Second review of de Meeus and Nous (2023): ***A new and almost perfectly accurate approximation of the eigenavlue effective population size of dioecious populations: comparisons with former other estimates and detailed proofs***

The authors have extensively revised their manuscript and have at least acknowledged and discussed all of the major questions that I raised concerning the original submission. My overall assessment of the work has not changed very much. I agree with the authors that there is intrinsic value in having more accurate approximations for theoretical quantities such as the eigenvalue effective population size of a dioecious population. It is also useful to have explicit derivations for these other approximations, especially when these are accompanied by biological interpretations for the different terms that appear in these expressions. At the same time, I suspect that the relatively small differences between these various approximations will be swamped by the errors arising from using models that ignore demographic complexities such as population dynamics, population structure and complex life histories. I also remain somewhat skeptical of the value of methods-of-moments estimators for N_e, particularly when these require the exclusion of otherwise informative data. I understand that implementing Bayesian and likelihood-based estimators is non-trivial, but these approaches are more statistically sound and should make more efficient use of all of the available data.

Since it would require extensive work, far beyond the scope of the current manuscript, to fully address either of these criticisms, I am satisfied with the additional text included by the authors discussing the limitations of their work. There is much more to be said about the relationship between genetic drift and demographic processes, but this manuscript does make some useful and interesting contributions. See below for a list of suggested grammatical and textual corrections.

Jay Taylor (9 May 2023)

Minor corrections:

line 2:"population size of a dioecious population: comparisons with other estimates and detailed proofs"

line 23: "the eigenvalue effective population size of a dioecious population"

line 26: "provides more accurate results in very small populations"

lines 37-45: I would be careful to distinguish between the Castle-Weinberg model and the Fisher-Wright model: the latter includes the assumption of binomial sampling in a finite population, whereas the former assumes an infinite population where genetic drift can be neglected. In my opinion, it is binomial sampling that is the core assumption of the Wright-Fisher model and it is in terms of this particular probabilistic model for parent-offspring relationships that the various concepts of effective population size are defined.

line 63: "In this note, we review some of these results and we then derive a new and apparently more accurate approximation for the eigenvalue effective population size of a dioecious population."

line 70: "can easily access this knowledge."

line 126: “from the pool”

line 128: “the probabilities of identity”

line 265: “In Figure 1”

Figure 1: Please specify the actual sex ratio(s) used to obtain the results shown in the figure on the left (uneven sex ratio).

line 329: “Notice that equation 20 differs from equation 4 by the subtraction of half an individual.”

line 372: “where only dioecy”

line 373: “2) to present detailed derivations of both the old and new results that would be accessible to most readers”

line 384: “with not much harm”

line 472: “We may also bear in mind that although random mating was assumed, we did not specify any reproductive strategy”

line 495: “F_{IS} should be estimated from adults”

General formatting suggestion: New lines should only be indented if they appear at the beginning of a new paragraph and not simply because they are preceded by an equation. Otherwise, the text becomes fragmented into numerous paragraphs containing only one to two sentences apiece (plus an equation), which reduces the coherence of the writing. It is perfectly acceptable for one or more equations to appear within a paragraph, provided the surrounding text addresses one or a few closely related ideas.