Comments on "Density dependent environments can select for extremes of body size" by Tim Coulson, Anja Felmy, Tomos Potter et al.

Major comments

1. **Consistency in notation of parameters**. There is some inconsistency in using letters with subscripts throughout the manuscript. For example, on page 6 there are parameters with subscripts in italics (α_N). The same parameter has no italics in the Table 1. Usually, it is up to the authors which symbols and letters to use for model parameters and variables. I would use no-italics subscripts everywhere and left italics to denote a parameter/variable that has a particular numerical value. However, as I said, it is up to the authors. The only think I would ask is to be consistent and stick to one chosen strategy.

2. **Recruitment vs Reproduction**. Through the entire manuscript, the authors use the term "reproduction". However, in the Table 1, there is another term used – "recruitment" – describing the same notations (ρ and R). I believe that both terms might describe similar processes, although not the same. For me, "reproduction rate" could be either number of offspring produced or number of copulations (both per-capita). "Recruitment" would describe number of juveniles reaching reproductive state. While there might be some discussion what either term means, I would recommend to use only one of them, stating clearly in the text what the authors want to say with it. Using both terms confused me a little when I was reading the paper and referring to the Table with the list of parameters.

Minor comments

- Line 34. It feels like a citation is needed after "There are clearly physiological limits…"
- Line 35. "… but the factors that push organisms…" Did you mean evolutionary factors here? Or factors, acting on the evolutionary time-scale?
- Line 39. "In contrast, as size increases, the values …" It is a nice statement. However, it might be good to provide a citation here, because it is not entirely clear whether you still refer to sources mentioned earlier in the text.

- Lines 56-59. You listed here a set of factors able to generate trade-offs. Can we consider here time as a limiting factor? Individuals may have enough resources but limited time to gain, incorporate or use resources.
- Line 59. "Trade-offs consequently arise…". Here I would mention that trade-offs arise on an individual level (a particular individual meets resource/time/space constrains), that result in population growth limits
- Line 89. "…fitness is carrying capacity…". This is, probably, the most subjective comment. In most cases, I prefer using the phrase "X is a proxy for fitness". Considering the variety of modelling approaches, there are several ways how one can define fitness. But in all the approaches, there is always a proxy for fitness.
- Line 101. "… would be less than unity." Unity = one?
- Line 106. "Since we know what…" I would recall here carrying capacity.
- Lines 112-124. This part seems somehow not in the right place. You describe well two scenarios how density-dependence acts later in the Methods. Also, you explain nice results for each of the scenarios. With this, your discussion and conclusion look logic. In contrast, mentioned part here is perceived a bit dry. I would either add some examples (for instance, after the third paragraph), or move this part of the text entirely at the end of the manuscript.
- Line 132. Extra closing bracket in the denominator of the equation.
- Line 145. One can deduce what each capital letter (but S) from the equation means. However, recalling meaning of letters would increase understanding
- Lines 147-150. This is one of the most crucial parts of the Methods, where you describe two scenarios. It will not hurt if you write here sentences in a more pro-active way. For example, one could phrase "In the first scenario, density dependence acts on reproduction, limiting number of offspring produced. We modelled this assuming $\rho < 0$. In the second scenario, population size is controlled through juvenile survival…". By no means I want to force you rewriting this part of the methods. I simply believe that pro-active phrasing makes the text much more attractive, especially when you describe your model assumptions.
- Line 151. Unity=one?
- Line 156-159. Just to clarify. Here you mentioned 20 models for each of the scenarios. Models represent different life histories. Later, you say that all but two functions (development and size at maturity) are identical. Does this mean that difference in life histories is based on these two functions?

- Lines 197-199. There are several terms provided (i.e. $\rho_N K$). It will increase understanding if you could provide or recall a biological meaning of all these terms.
- Line 254. Can you recall briefly, what the equation here stands for?
- Line 286. "… was highly non-linear…". Can non-linearity be of a particular level? Is it measured somehow?
- Line 306. It might be helpful to explain briefly what the threshold means. You refer to the figure, where one can understanding what a threshold is. But, the description is lacking here.
- Line 323. I am confused with phrasing "Below the threshold of minimum fitness…". Does it mean that every life-history strategy that is below this threshold is not selected at all? It would be great if you could clarify here. The same comment relates to Line 306
- Line 348. What is the biological meaning of these equations?

Table 1
1. It might be useful for readers to have some information about ranges of parameters and variables (where possible) used in the model. This can be done by adding an additional column. There is table 2 with (presumably) values for most of the parameters. If so, it might be good to refer table 2 here
2. Recruitment vs Reproduction comment (see above). There are recruitment terms in the table, while reproduction in the text

Table 2
1. I would add a few more lines describing this table. For example, mention that all these parameter values are the same across all the models you constructed (according to what you wrote in lines 156-157). In addition, it would help readers to see connection if you provide parameter notations from Table 1.
2. There are no rows indicating values for the development function and the size at sexual maturity, which are unique for every model (according to lines 156-157)
3. What the empty space and tree dots under "Growth parameter" stand for?

Figure 1
1. Fig1A – What do black lines mean? If they are necessary and bring some useful information, can you describe them?

Figure 2

1- "Vertical lines separate juveniles from adults." – I would assume that juveniles are on the left side of the graphs (C) and (D). However, there is inconsistency in color use for slow life history strategy (shades of red). On the (C) graph yellowish color indicates (supposedly) adults (right side from the vertical line). While on the (D) graph the same shade indicates the left part of the area under the curve.

2- (D). "However, because juvenile and adult size…". I am confused by this sentence. I would assume you meant that survival rates are higher than for the faster life history, considering what we can observe on the Fig 2D. But I may be wrong. Can you clarify this part?

3- Survival rates are indicated with pastel dots on Fig 2C, while with bright dots on Fig 2D. It might be good to be consistent how you picture the same parameter.

Figure 3

1. Vertical green line. From these plots and your description, it is clear that this line shows indeed the least fit life history. However, from the text (see my comments above – line 306 and line 323), one could assume that this is a threshold separating the fittest life histories from less fit.

2. What does the x axis label at Fig 3F mean? Could you add a brief explanation in the figure capture?

Figure 4

1. Fig 4A and B. It is a nice idea to add a dashed line to Fig 4B for better comparison. In such a case, I would not mention Fig 4A at all, because it might be hard to compare lines on separate plots. Even if there is some difference, it can be unnoticeable.

Figure S1

1. From the text, one can get that these plots depicts scenario 1 (am I right?). But there is no notation in the figure capture.

2. What does a green dot indivate?

3. I am confused with the description of "fast vs slow" life histories. In the text, you wrote about the positive covariance between parameters determining the evolution of fast life histories (lines 279-280). Do these histories evolve from green dots? Evolving from green dots, are these histories stay "fast-life"? Based on your figure legend, one may assume that not – "Blue dots represent fast life histories, red dots slower ones".

Figure S2

1. Do you need graphs E and H? I would say that they are rather not necessary.

2. Same comment on the green dots and mixing colors to picture both "slow-vs-fast" life histories and fitness of strategies